Nonlinear Interaction of an Electromagnetic Wave with a Plasma Layer in the Presence of a Static Magnetic Field. IV. Experimental Results*

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The nonlinear interaction of an electromagnetic wave with a uniform, weakly ionized anisotropic plasma layer has been investigated experimentally. The experiment was designed to comply as closely as possible with the assumptions used in the theory of the nonlinear interaction, which is given in parts I, II, and III of this series of papers. Experimental results on sum and difference frequency mixing and harmonic generation are described as a function of the ambient electron density, the electron-neutral particle collision frequency, the external dc magnetic-field strength, thickness of the plasma layer, and the field strengths and frequencies of the incident waves. The case of linearly polarized, small-amplitude microwave signals incident on a layer of helium plasma is examined. Within the plasma, the signals propagate as plane waves with their directions of propagation and polarization normal to the dc magnetic field (i.e., the extraordinary mode of propagation). A detailed comparison is made between the experimental results and the theoretical predictions. Quantitative agreement was obtained over the electron density range permitted by the experiment. The results show that the plasma model assumed in the theory is an accurate representation of the actual plasma, and that the small-signal analysis accurately predicts the effects of the nonlinear terms in the Boltzmann equation on propagation phenomena. However, special care is required to insure that the plane-wave, infinite-medium assumption employed in the theory is satisfied experimentally. The experimental results also establish the validity of using certain characteristics of the nonlinear phenomena as a tool for measuring the electron density in a plasma as proposed in II and III. A resonance was detected in the sum frequency wave which is not predicted by the theory. This resonance occurs for values of the dc magnetic-field strength corresponding to cyclotron resonance at the arithmetic mean of the two incident frequencies. The origin of this resonance is not understood at this time.

INTRODUCTION

ONE of the consequences of the nonlinear nature of the interaction of an electromagnetic wave with an ionized medium is the generation, within the plasma, of different frequencies from those incident upon the plasma. For example, a wave of angular frequency ω impressed upon a plasma can generate waves of frequency $m\omega$ (m a positive integer), and if two waves of frequencies ω_1 and ω_2 are simultaneously impressed upon a plasma, waves of combination frequencies $\omega = m\omega_1 \pm n\omega_2$ (m and n integers) can be generated.

Most of the experimental investigations of the frequency conversion properties of plasmas have been concerned primarily with harmonic generation.[1] However, Dreicer[2] observed the generation of combination frequencies in a simulated free-space plasma which he attributed to the interaction between an rf electric field and electron density gradients in the plasma. Using $\omega_1/2\pi = 28$ Mc/sec and $\omega_2/2\pi$ near 3 Gc/sec and near 10 Gc/sec, he detected frequencies $\omega_2 + n\omega_1$, where n ranged from 1 through 36. Most of the experiments referred to were performed in geometries and under conditions for which it was difficult to make a quantitative comparison between the data and theory. However, Baird and Coleman[3] analyzed and observed frequency conversion in a microwave discharge between parallel plates in a waveguide, and obtained excellent agreement between theory and experiment. The nonlinear mechanism they postulated was that of a modulation of the electron density at the fundamental and harmonic frequencies, due to a modulation of the ionization frequency of the plasma, by a very intense incident electromagnetic wave. In their experiment a magnetic field was sometimes used, but only as a means of confining the discharge so as to improve the efficiency of conversion.

The present work describes the results of an experimental investigation of the nonlinear interaction of small-amplitude electromagnetic waves with a magnetoplasma layer. The theoretical portion of this work has been described in part I,[4] part II,[5] and in the preceding paper, part III,[6] hereafter referred to as I, II, and III, respectively.

EXPERIMENTAL APPARATUS AND TECHNIQUES

The experimental apparatus and techniques are designed to approximate as closely as possible the assumptions made in the theoretical analysis. A low-pressure helium plasma was created in a large rectangular parallelepiped Pyrex container located in a uniform dc magnetic field. Linearly polarized plane waves, at frequencies near 9 and 18 Gc/sec were transmitted into the plasma slab and propagated in the extraordinary mode within the plasma. A schematic representation of

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† Portions of this work were performed while the authors were members of the former Palo Alto Laboratories, General Telephone and Electronics Laboratories.

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[2] H. Dreicer, Bull. Am. Phys. Soc. 7, 151 (1962).

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[5] R. F. Whitmer and E. B. Barrett, Phys. Rev. 125, 1478 (1962).

[6] E. B. Barrett, R. F. Whitmer, and S. J. Tetenbaum, preceding paper, Phys. Rev. 135, A369 (1964).

the experimental arrangement is shown in Fig. 1. The walls of the discharge vessel were ground to a thickness of a half wavelength at 9.15 Gc/sec in order to reduce reflections from the pyrex boundaries. A typical vessel had an inside dimension, in the direction of propagation, of approximately 3 free-space wavelengths at 9.15 Gc/sec, and inside transverse dimensions of approximately 4 free-space wavelengths. After heating to 400°C, the vessel was evacuated to a residual pressure below 10^{-9} Torr and filled with Linde spectroscopically pure helium at a pressure in the range from 10^{-3} to 1 Torr. A series of calibrated vacuum gauges were used to measure the gas pressure in this range. The plasma was produced by placing the discharge vessel between the plates of a capacitor which formed part of the external-tank circuit of a 21-Mc/sec generator. An electron density of approximately $10^{12}/\text{cm}^3$ could be produced with 500 W of cw power from this generator. Two different electromagnets, each with a 9-in. gap spacing were used. One magnet had 12-in.-diam pole faces and was specially designed to provide a field uniformity of 0.5% over the discharge volume in the range from 0 to 4000 G. The other magnet had 16-in.-diam pole faces and produced fields from 0 to 7200 G. Its uniformity was approximately 1.5% over the discharge volume. An automatic magnet current controller swept the magnetic field through the desired range.

The incident cw microwave signals were frequency stabilized and propagated in the extraordinary mode unless otherwise stated. Second harmonic measurements were made for incident frequencies near 9 Gc/sec and near 18 Gc/sec. The sum frequency experiments utilized two different incident frequencies near 9 Gc/sec while the difference frequency experiment utilized one incident frequency near 18 Gc/sec and the other near 9 Gc sec. When measuring the transmitted second harmonic or transmitted sum frequency waves, standard rectangular waveguide components were usually employed. However, single-ridged waveguide components[7] were occasionally used to allow the simultaneous transmission and/or reception of both 9- and 18-Gc/sec signals through the same plasma path. A further use of the ridged waveguide was in propagating an 18.2-Gc/sec interferometer signal through the plasma. This allowed the measurement[8] of the average electron density in the same region of the plasma within which the frequency conversion process was occurring. To insure plane-wave propagation, dielectric lenses were used with rectangular waveguide horns on both sides of the discharge vessel. Each lens was followed by an aperture cut in a microwave absorbing sheet in order to reduce diffraction effects. Additional absorbing material was placed around exposed metal surfaces in order to reduce reflections from these surfaces. The aperture was approximately 4 free-space wavelengths in diameter.

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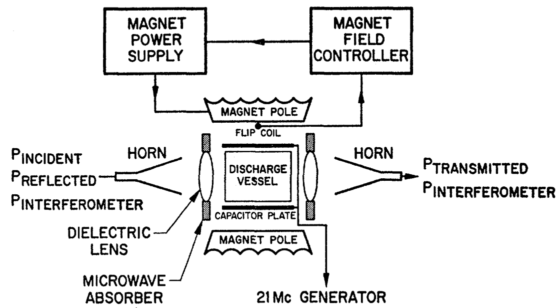

FIG. 1. Diagram of exprimental arrangement.

Phase and amplitude patterns of the horns in both E and H planes were measured under conditions similar to that of the actual experiment, but with no discharge present. Phase variations were less than a tenth of a wavelength over 80% of the discharge volume. The amplitude patterns were such as to effectively illuminate only the central region of the discharge vessel, thereby reducing the effects of plasma nonuniformities and scattering from the side walls of the vessel.

Sensitive superheterodyne receivers were used to detect the generated second harmonic and combination frequency waves. To guarantee that a particular detected wave was actually being generated by the plasma, appropriate microwave bandpass, bandstop, high-pass and low-pass filters were incorporated into the input and output waveguides. The receiver output was fed to a logarithmic converter and thence to an XY recorder. The recording gave a plot of the amplitude of the desired generated signal as a function of the static magnetic field.

EXPERIMENTAL RESULTS

Several tests were conducted in order to determine whether the experiment conformed to the assumptions of the theory. An indication of the degree to which the plasma slab satisfied the assumption of infinite transverse dimensions was obtained in two ways. First, by measuring the fundamental, second-harmonic, and mixed-frequency waves emerging from the plasma in directions normal to the direction of propagation, it was found that the power scattered in these directions was at least 25 dB down from the corresponding powers along the axis of propagation. Second, measurements of the second harmonic wave generated by an incident signal at 18.3 Gc/sec, for which the transverse dimensions are 8 free-space wavelengths, gave results similar to those for an incident signal at 9.15 Gc/sec. Both tests indicate that the infinite-medium assumption was reasonably satisfied.

The uniformity of the plasma under typical operating conditions was investigated by means of a microwave interferometer and also by examining the 6678 Å light output and the total light output of the helium plasma. The microwave and optical data were consistent with

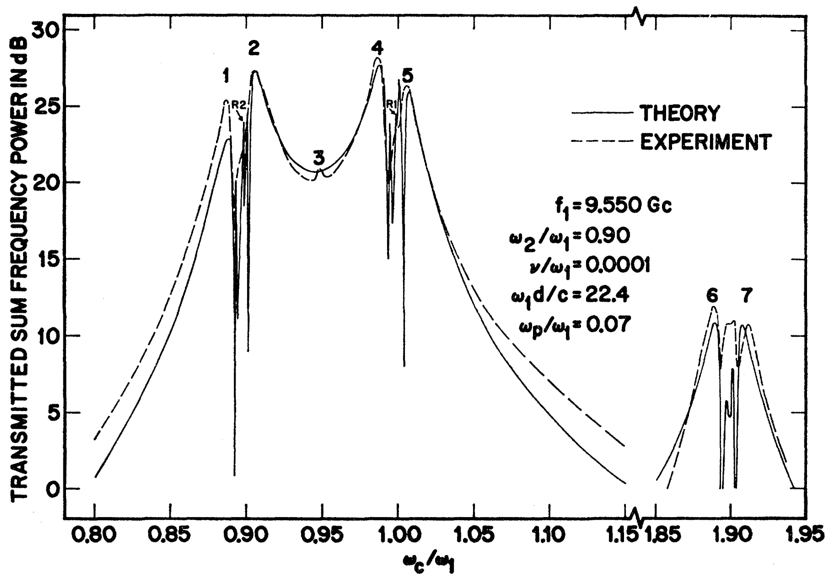

FIG. 2. Transmitted sum frequency power versus ω_c/ω_1.

each other. It was found that for fixed values of the magnetic field, the electron density was uniform within 20% over an 8-cm-diam disk in any plane normal to the direction of propagation. The electron density in the longitudinal direction remained constant to within 20% except in the immediate vicinity of the front and rear walls. For a fixed region of the plasma being probed, the electron density remained constant within 20% as the magnetic field was varied from 1500 to 6500 G. The effective temperature of the plasma is not accurately known. Double-probe measurements made in a similar discharge arrangement indicate a temperature of a few electron volts.

The amplitudes of the generated harmonic and combination frequency waves exhibited a complex resonance behavior as a function of magnetic field as predicted by the theory. It was found that the shape of the resonance curves was a sensitive function of the microwave optics used and care was required in order to satisfy the plane-wave, infinite-medium assumption used in the theory. Placing the transmitting and receiving apertures close to the discharge vessel provided a means for obtaining a reasonable approximation to this assumption.

The theoretical analysis makes use of a small-signal approximation, as defined in II. Experimentally, one can vary the incident power and determine at what power levels the resonance curves begin to change shape. For cw powers up to approximately 0.5 W into the transmitting horns, no significant changes could be observed in the shape of the resonance curves. Unless otherwise indicated, incident powers were kept below this level.

In the experimental data to be discussed below, certain values of ν, the electron-neutral particle collision frequency, and ω_p, the plasma frequency, are specified. The values of ν were calculated from the pressure p, in Torr by use of the relationship[9] $\nu = 2.5\times10^9 p$. The values of ω_p can be obtained from the interferometer measurements mentioned above. However, it was shown in III that a sensitive measurement of the electron density can be obtained by measuring the values of the magnetic field for which the transmitted sum frequency power exhibits geometrical resonances. This technique was used to obtain ω_p. The interferometer measurements gave values for ω_p approximately 10% higher than those obtained from the sum frequency curves.

Sum Frequency Mixing

The sum frequency mixing experiment was performed by propagating two cw signals near 9 Gc/sec along the identical propagation path into the plasma and detecting the transmitted and reflected sum frequency waves at a frequency near 18 Gc/sec. Measurements were made of the relative sum frequency power in dB, P_S, as defined in III, as a function of applied magnetic field for a wide variety of plasma and incident-wave conditions.

Figure 2 shows a comparison between a recording of the transmitted sum frequency power and the corresponding theoretical curve. The peaks in the experimental curve, designated 1, 2, 4, 5, 6, and 7 correspond to the geometrical resonances, and $R1$ and $R2$ to the plasma resonances of Fig. 1 of III. The experimental curve was normalized to the theory so that the amplitudes of peak 2 were equal. The portion of the experimental curve in the region near cyclotron resonance for the sum frequency wave was corrected for the cyclotron emission from the plasma at the sum frequency. The

[9] This relationship is approximately correct for helium in the electron energy range for which ν is independent of electron energy. In the pressure range of this experiment, a factor of 2 error in ν has little effect on the results.

peak cyclotron emission amplitude was approximately 10 dB less than the true generated sum power, and its contribution to the total detected power was subtracted out.

A small peak, designated 3, is discernible between peaks 2 and 4. This peak is not predicted by the theory. At a pressure of about 3×10^{-3} Torr, at which the data of Fig. 2 were taken, peak 3 is quite small. As the pressure is increased, however, the amplitude of peak 3 increases relative to that of the other resonances. At $p=2\times10^{-2}$ Torr it is much more pronounced, as can be seen from Fig. 3. In the figure, the amplitudes of the two curves were normalized at peak 1. The agreement between theory and experiment is good except for the appearance of peak 3. Recordings were also made of the reflected P_S versus B_0 curves and showed good agreement with the corresponding theoretical reflected sum frequency curve shown in Fig. 1 of III except for the occurrence of peak 3.

A series of recordings was taken of the transmitted P_S for different values of ω_p. The positions of the resonance peaks, as a function of magnetic field, are plotted in Fig. 4 and compared with theory over the electron density range measured. Qualitative agreement between theory and experiment was observed for values of ω_p/ω_1 up to about 0.4. The data for the $R1$ peak are very similar to that of the $R2$ peak shown, and were not plotted. For $\omega_p/\omega_1<0.05$, peaks 1 and 2 coalesce as do peaks 4 and 5. The position of peak 3 is also plotted indicating a very small dependence on ω_p. For values of ω_p/ω_1 beyond about 0.18, peaks 2 and 4 merge. As can be seen from the figure, the positions of the peaks are sensitive functions of ω_p/ω_1 and therefore this can provide an accurate means for measuring the electron density.

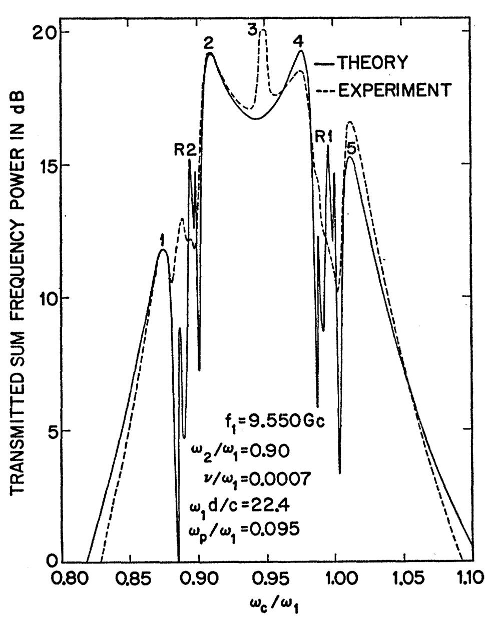

FIG. 3. Transmitted sum frequency power versus ω_c/ω_1 near resonance regions.

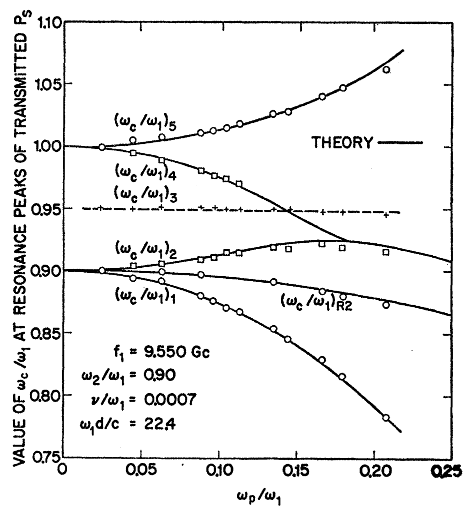

FIG. 4. Value of ω_c/ω_1 for resonance peaks of transmitted P_S versus ω_p/ω_1.

The experimental amplitudes of several of the resonance peaks of the transmitted P_S, as functions of electron density, are compared with theory in Fig. 5. The data pertain to the same plasma for which the data of Fig. 4 were taken. The theoretical curves were normalized to provide the best fit to the experimental data for peak 1. It should be noted that the amplitude of peak 3 exhibits a somewhat different characteristic from the amplitudes of the other peaks as ω_p is varied.

To determine quantitatively the dependence of the sum frequency power on P_1 and P_2, the plasma parameters, magnetic field and incident frequencies were fixed and P_1 was varied from 10 mW to 4 W for two different values of P_2. The results are shown in Fig. 6. It is seen

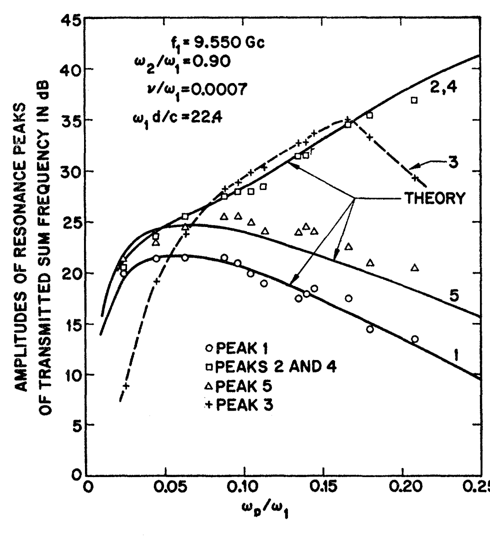

FIG. 5. Amplitudes of resonance peaks of transmitted P_S versus ω_p/ω_1.

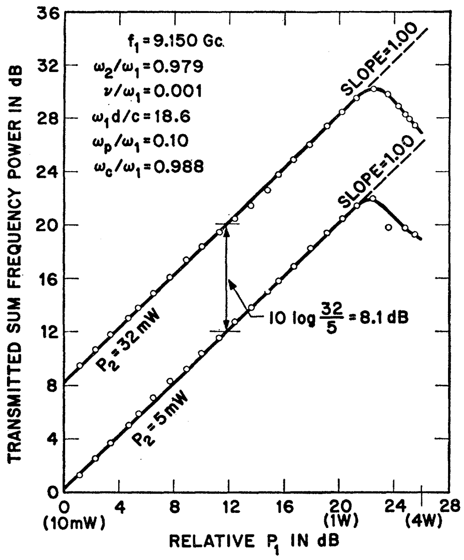

FIG. 6. Transmitted sum frequency power as a function of the incident power.

that for P_1 below about 1 W, P_S is directly proportional to P_1. The two straight lines are separated by 8.1 dB which corresponds exactly to the difference in the values of P_2, and hence, implies that P_S is directly proportional to the product of P_1 and P_2 as predicted by the theory. For incident powers greater than 1 W, P_S decreases. A microwave breakdown in the plasma occurred at these powers resulting in pronounced changes in the electron density and possibly changes in the collision frequency. The absolute value of the experimental transmitted power agreed with the theoretical value within experimental error, and the difference in amplitude between the experimental transmitted and reflected P_S waves agreed very closely with theory.

Since the generation of the second harmonic is the limiting condition of sum frequency generation as the two incident frequencies approach one another, it is of interest to examine the positions of the resonances of P_S as ω_2/ω_1 approaches 1.0. These positions are plotted in Fig. 7. Except in the immediate vicinity of $\omega_2/\omega_1=1$, the positions of peaks 1, $R2$ and 2 line on straight lines of slope 1 while the positions of peaks 4, $R1$ and 5 lie on straight lines of slope zero. This behavior is as expected since the positions of the one set of peaks depend only upon the wave of frequency ω_2, and the positions of the other set of peaks depend only upon the wave of frequency ω_1. As ω_2 approaches ω_1, peaks 1 and 5 go over smoothly into the corresponding peaks of the second harmonic wave, and peaks $R2$, 2, 4, and $R1$ all merge into the major single resonance peak of the second harmonic wave. The positions of the experimental peak 3 are also plotted. These lie on a straight line of slope $\frac{1}{2}$. One would expect this behavior if its position depended only upon the arithmetic mean of the two frequencies. Extending this line to $\omega_2/\omega_1=1$ yields a position of $\omega_c/\omega_1=1.00$ for peak 3, in agreement with the experimental curves for $P_{2\omega}$, to be presented later. The experimental positions of the major peaks in the reflected P_S wave were also investigated as functions of ω_2/ω_1. It was found that peaks occur at the same magnetic fields as the $R2$ and $R1$ peaks of the corresponding transmitted P_S curves. The experimental amplitudes of the resonances in the transmitted P_S versus ω_2/ω_1 curve were also examined and were in agreement with the theory.

The manner in which the P_S curves vary with ν was also investigated and the results are in qualitative agreement with the theoretical predictions of III. The linewidths of all of the resonances except 3 increase with increasing ν/ω_1 and also with increasing ω_p/ω_1. The linewidth of peak 3 also increases with increasing ν/ω_1, but is essentially independent of ω_p/ω_1. It is probable that the observed linewidths of the narrower resonances differ somewhat from the true linewidths because of the magnetic-field inhomogeneities.

So far, we have discussed the case where both incident waves propagated in the extraordinary mode, with the same direction of propagation. Experiments were performed in which the directions of propagation of the incident waves remained the same, but the polarization of one wave was varied with respect to the other. It was found, as expected, that only the electric-field components of the incident waves perpendicular to B_0 contribute appreciably to the generation of sum frequency power.

Experiments were also performed in which both incident waves were propagated normal to the direction of the magnetic field, but at right angles to each other. Again, only the electric-field components of the incident waves which are perpendicular to B_0 contribute appreciably to the sum frequency power generation. The amplitude of the generated sum frequency power was reduced by more than 20 dB compared to that generated when both incident waves propagate in the same direction. The shape of the curve of the sum fre-

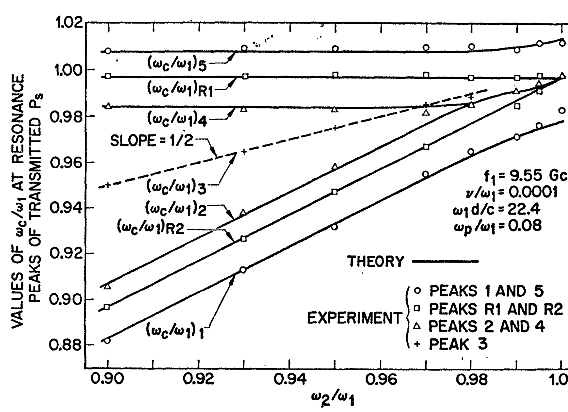

FIG. 7. Values of ω_c/ω_1 for resonance peaks of transmitted P_S versus ω_2/ω_1.

quency power versus B_0 was similar to that previously found. The amplitudes of the peaks which occur near plasma resonance for the incident wave which propagates in the direction of the receiving horn were enhanced relative to the amplitudes of the peaks which occurred near plasma resonance for the other incident wave, as would be expected. In all the experiments, the detected sum frequency wave was polarized in a direction perpendicular to B_0.

Experiments were performed to study the nonlinear interaction between the incident microwaves and the RF excitation producing the plasma, as suggested by the work of Dreicer.[2] The experiments consisted of searching for waves of frequency $\omega+m\omega_{RF}$. Using a microwave signal polarized transverse to B_0 with a frequency $\omega/2\pi=9.15$ Gc/sec and $\omega_{RF}/2\pi=21.6$ Mc/sec, transmitted waves were detected at frequencies corresponding to integral values of m from -16 to $+21$. These combination frequencies were generated in an air plasma in the pressure range from 10^{-5} to 3×10^{-2} Torr with microwave powers below 1 W and rf powers below 20 W. The magnetic field dependence of the sideband waves was similar to that for the microwave-mixing experiment and hence, their origin is believed to be similar to the origin of combination waves whose generation is discussed in III. This explanation is somewhat different from Dreicer's explanation which is based upon electron density gradients. The sideband power was directly proportional to the microwave power in the range from 1 mW to 1 W. It varied in a rather complicated way with the excitation power, but in general increased with increasing excitation. For excitation powers below 10 W, the first sidebands were typically about 75 dB down from the incident microwave power. The sideband amplitudes tended to decrease with increasing m. Since the effects of the rf excitation were easily discernible, it is believed that they did not contribute to any of the P_S measurements. In particular, the rf excitation appears to have no connection with the appearance of peak 3.

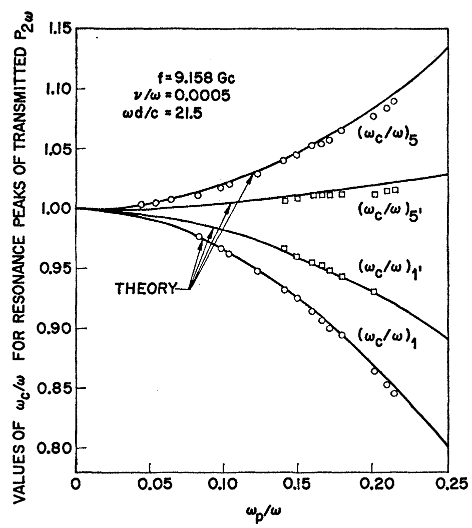

FIG. 9. Value of ω_c/ω for resonance peaks of transmitted $P_{2\omega}$ versus ω_p/ω.

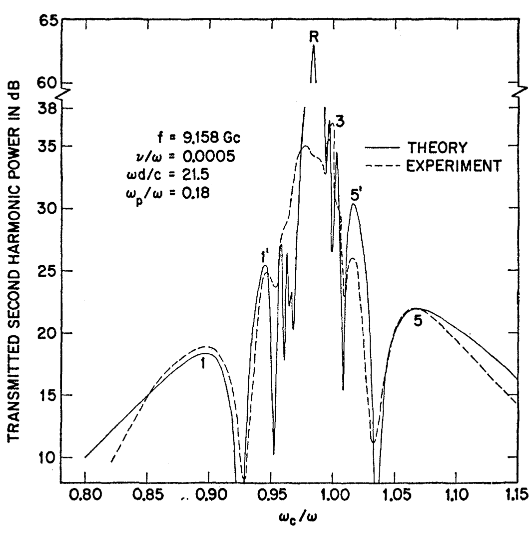

FIG. 8. Transmitted second-harmonic power versus ω_c/ω.

Second-Harmonic Generation

Second-harmonic-generation experiments were performed using both the 12- and 16-in. pole-face-diam electromagnets. The former gave more precise measurements because of the better uniformity. The wider magnetic-field range of the latter was required to observe the resonance region around $\omega_c/\omega=2$ for an incident wave near 9 Gc/sec. Figure 8 shows a typical plot of the transmitted second harmonic power $P_{2\omega}$ versus ω_c/ω in helium. The corresponding curve (from I) exhibits two outer, relatively broad geometrical resonances designated peaks 1 and 5, and one central, major resonance peak designated R. A number of narrower geometrical peaks appear between peaks 1 and R and R and 5, the one closest to peak 1 being designated peak 1′ and the one closest to peak 5 being designated 5′. The amplitudes of the curves were normalized to provide equal amplitudes for peak 5. The experimental curve shows the presence of most of the major theoretical peaks. The peak labelled 3 should not be confused with the geometrical resonance shown on the theoretical curve. Peak 3 is located near $\omega_c/\omega=1.0$ and its position is independent of ω_p/ω. This peak corresponds to peak 3 of P_S as ω_2/ω_1 approaches 1.0.

The amplitude of the experimental peak R is more than 25 dB below the theoretically predicted peak amplitude. The reason for this is not definitely known. However, for the low pressures used, the theoretical linewidth of peak R is very small since $(\Delta\omega_c)_{3\mathrm{dB}}=2\nu$ as shown in II. Since small spatial variations of the electron density and the dc magnetic-field strength do exist within the plasma volume, the exact resonance

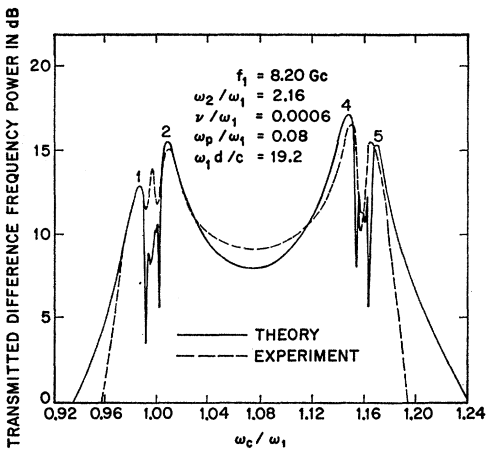

FIG. 10. Transmitted difference frequency power versus ω_c/ω_1.

relations can only be satisfied over a small portion of the plasma. Hence, one would expect a broadening and a reduction in the amplitude of the resonance peak. Calculations of these effects indicate that an amplitude reduction of 20 dB or more is possible in our experiment. An experiment in which the magnetic-field inhomogeneity was deliberately increased verified that peak R was further broadened and reduced in amplitude as the magnitude of the inhomogeneity was increased. The inner geometrical peaks between $1'$ and R and between R and $5'$ are masked, also probably due to the nonuniformities mentioned above.

The variation of the positions of the peaks of $P_{2\omega}$ as a function of the electron density was also studied. The data are plotted in Fig. 9 and compared with theory. The position of the major peak R is not plotted in Fig. 9 because it was not possible to accurately determine its position. Nevertheless, the central resonance region corresponding to peak R was found to shift to lower magnetic fields as ω_p increased, in qualitative agreement with the theory. The transmitted $P_{2\omega}$ in the vicinity of $\omega_c/\omega=2$ was also recorded and showed agreement with theory. Measurements were also made of the amplitudes of the peaks of $P_{2\omega}$ as a function of ω_p. Reasonable agreement with theory was found except for the amplitude of the main resonance. Since the data are similar to that presented for P_S they are not shown here. In addition, the difference in amplitude between the transmitted and reflected $P_{2\omega}$ waves was also in agreement with theory.

The theory predicts that for fixed incident frequency, magnetic field, and plasma parameters, the generated second-harmonic power should be directly proportional to the square of the fundamental power P_ω. Measurements indicate that for incident powers between 1 mW and 1 W, $P_{2\omega}$ is directly proportional to $(P_\omega)^{2.0}$. Above 1 W of incident power, microwave breakdown occurred near the front face of the discharge vessel, and, as in the case of P_S, marked deviations form square-law behavior were observed.

Difference Frequency Mixing

The difference frequency experiment was performed in the same manner as the sum frequency experiment except that incident waves at frequencies near 8 and 18 Gc/sec were used. In order to provide magnetic fields extending beyond cyclotron resonance for the higher frequency incident wave, the 16-in. electromagnet was required. Figure 10 shows a comparison between a recording of the transmitted difference frequency power and the corresponding theoretical curve for P_D as defined in III. There are two resonance regions shown in the figure, one near $\omega_c/\omega_1=1$ and the other near $\omega_c/\omega_1=1.16$, or $\omega_c/(\omega_2-\omega_1)=1$. A third resonance region occurs near $\omega_c/\omega_2=1$, but is not shown. The amplitudes of the two curves were normalized at peak 1. The shape of the experimental curve in the region near $\omega_c/\omega_1=1.16$ was not recorded directly due to cyclotron emission in this region as in the case of P_S. Experiments were performed to determine the power dependence of P_D on P_1 and P_2 and the effects on P_D of changing the incident frequencies, the electron density and the collision frequency. The results were all in qualitative agreement with the theoretical predictions of III. Since these results are very similar to those presented for P_S, they are not presented here.

Theory predicts that the amplitude of the transmitted difference frequency wave is approximately 15 dB less than the corresponding amplitude of the sum frequency wave. This result was experimentally verified.

CONCLUSIONS

A detailed experiment has been performed on the microwave frequency-conversion properties of an anisotropic helium plasma layer. Except as indicated below, quantitative agreement between the experimental results and the theoretical predictions was obtained, indicating that the assumptions made in the theory provide an accurate model of the actual plasma. In addition, the small-signal analysis, utilizing an iterative procedure for solving the nonlinear Boltzmann transport equation, accurately predicts the frequency-conversion properties of the plasma, and yields the amplitudes of the waves at the combination frequencies as functions of the plasma and incident wave parameters. The range of values of the incident power density for which the small-signal analysis is valid was discussed theoretically in II. The experimental results indicate that the incident power density becomes sufficient to cause microwave breakdown within the plasma before this range is exceeded. Also, a method was established for determining the electron density in a bounded magnetoplasma by measuring the magnetic-field strength at which a particular resonance of one of the waves at the combination frequencies occurs.

Two discrepancies between the theory and experiment were found. First, the amplitude of the major plasma resonance peak in the second-harmonic power was 20 dB smaller than predicted by the theory. This is probably attributable to the nonuniformities in the electron density and dc magnetic-field strength in the plasma which reduce the resonance amplitude. The effects of these nonuniformities on the narrow plasma resonance peaks are naturally more pronounced than on the broad geometrical resonances. Second, an additional resonance, not predicted by the theory, was discovered in the combination frequency power as a function of the dc magnetic-field strentgh. This resonance occurs near values of the dc magnetic field for which the electron cyclotron resonance frequency is equal to the arithmetic mean of the frequencies of the incident waves. Unlike the other resonance peaks, the magnetic-field value for which this resonance occurs is independent of the electron density. The origin of this resonance is not understood at this time.

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Motion of a Charged Particle in a Constant Magnetic Field and a Transverse Electromagnetic Wave Propagating along the Field

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(Received 24 February 1964)

The relativistic equation of motion is examined for a charged particle in a constant magnetic field and a transverse electromagnetic wave propagating along the field. A general dicussion is given of the effects at cyclotron resonance of the magnetic field of the wave and the relativistic mass increase with energy. An exact solution to the equation of motion is found for the case of a circularly polarized wave. The solution shows that when the index of refraction of the medium in which the wave propagates is not unity, the energy of the particle is a periodic function of time, the exact relationship being expressible in terms of elliptic integrals. When the index of refraction is unity, the effect of the magnetic field of the wave just compensates for the change in mass with energy, and the energy of the particle increases indefinitely at resonance. Several possible applications of this solution to classical cyclotron resonance phenomena are pointed out. As a numerical example, the case of an electron in a constant magnetic field of 1000 G initially at resonance with microwaves whose E field is 0.1 esu is considered.

I. INTRODUCTION

THE interaction between a charged particle and an electromagnetic wave in the presence of a constant magnetic field underlies several phenomena currently under investigation concerning the Van Allen particles,[1] plasma in the earth's magnetosphere,[2] and the diagnostics, heating, and confinement of plasma in the laboratory.[3] This interaction exhibits resonance effects when the wave frequency is at or near the particle's cyclotron frequency. In this paper we study the nature of the interaction when neither the magnetic field of the wave nor the relativistic mass change of the particle with energy are neglected, and we place special emphasis on the resonance effects.

The equation of motion of a particle of rest mass m_r and charge e in a constant magnetic field $\mathbf{B}_0$ is

$$\dot{\mathbf{p}}=e[\mathbf{E}+(\mathbf{v}/c)\times\mathbf{B}+(\mathbf{v}/c)\times\mathbf{B}_0]. \tag{1.1}$$

Here $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields of the electromagnetic wave, $\mathbf{p}$ the particle's momentum, $\mathbf{v}$ its velocity, and c the speed of light in vacuo. Gaussian units are used, and the dot signifies differentiation with respect to time. The electromagnetic wave is characterized by an angular frequency ω and a propagation vector $\mathbf{k}$, and in this paper we consider only the case where $\mathbf{B}_0$ and $\mathbf{k}$ are parallel and $\mathbf{k}$ and $\mathbf{E}$ are perpendicular, i.e., a purely transverse wave which propagates parallel to the constant field $\mathbf{B}_0$. For convenience, we take the direction of $\mathbf{B}_0$ and $\mathbf{k}$ to be the z direction. If the medium through which the wave propagates has an index of refraction n, then

$$n=kc/\omega=B/E. \tag{1.2}$$

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